

Q12 * Mathematik * Differenzieren und Integrieren

1. Bestätigen Sie durch Rechnung die angegebenen Ableitungsfunktionen!

- a) $f(x) = (2x - 3)^3 \Rightarrow f'(x) = 6 \cdot (2x - 3)^2$
- b) $f(x) = 2 \cdot \sqrt{x^2 + 3} \Rightarrow f'(x) = \frac{2x}{\sqrt{x^2 + 3}}$
- c) $f(x) = (x^2 + 3x) \cdot (x^2 - 1) \Rightarrow f'(x) = 4x^3 + 9x^2 - 2x - 3$
- d) $f(x) = \frac{2x}{x^2 + 1} \Rightarrow f'(x) = \frac{2 \cdot (1 - x^2)}{(x^2 + 1)^2}$
- e) $f(x) = \frac{1 + x^2}{2x} \Rightarrow f'(x) = \frac{x^2 - 1}{2x^2} = \frac{1}{2} - \frac{1}{2x^2}$
- f) $f(x) = \frac{\sqrt{2x+1}}{x^2 + 1} \Rightarrow f'(x) = \frac{1 - 2x - 3x^2}{(x^2 + 1)^2 \cdot \sqrt{2x+1}}$
- g) $f(x) = \frac{3x - 1}{\sqrt{x^2 + 1}} \Rightarrow f'(x) = \frac{3 + x}{(x^2 + 1) \cdot \sqrt{x^2 + 1}}$
- h) $f(x) = \frac{x^2 - 1}{x^2 + 1} \Rightarrow f'(x) = \frac{4x}{(x^2 + 1)^2}$
- i) $f(x) = -x + x \cdot \ln(x) \Rightarrow f'(x) = \ln(x)$
- j) $f(x) = 2x \cdot \ln(1 + x^2) \Rightarrow f'(x) = \frac{4x^2}{x^2 + 1} + 2 \cdot \ln(1 + x^2)$
- k) $f(x) = \ln\left(\frac{2x}{x^2 + 1}\right) \Rightarrow f'(x) = \frac{1 - x^2}{x \cdot (x^2 + 1)}$
- l) $f(x) = \ln(\sqrt{x^2 + 1}) \Rightarrow f'(x) = \frac{x}{x^2 + 1}$
- m) $f(x) = \ln(x \cdot \sqrt{x^2 + 1}) \Rightarrow f'(x) = \frac{2x^2 + 1}{x \cdot (x^2 + 1)}$
- n) $f(x) = 2x \cdot e^{x+1} \Rightarrow f'(x) = 2 \cdot (x+1) \cdot e^{x+1}$
- o) $f(x) = (x^2 - x) \cdot e^{2x} \Rightarrow f'(x) = (2x^2 - 1) \cdot e^{2x}$
- p) $f(x) = \frac{2 - e^x}{1 + e^x} \Rightarrow f'(x) = \frac{-3 \cdot e^x}{(1 + e^x)^2}$
- q) $f(x) = \frac{e^{2x+1}}{e^{2x} + 1} \Rightarrow f'(x) = \frac{2e^{2x+1}}{(e^{2x} + 1)^2}$



2. Bestimmen Sie (mit Hilfe von Aufgabe 1) den Wert der folgenden bestimmten Integrale!

- a) $\int_1^3 \frac{2x}{\sqrt{x^2 + 3}} dx$
- b) $\int_{-2}^{-1} \frac{x^2 - 1}{2x^2} dx$
- c) $\int_0^1 \frac{3 \cdot e^x}{(1 + e^x)^2} dx$
- d) $\int_1^e \ln(x) dx$
- e) $\int_0^1 (2x^2 - 1) \cdot e^{2x} dx$



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1.

a) $f(x) = (2x-3)^3 \Rightarrow f'(x) = 3 \cdot (2x-3)^2 \cdot 2 = 6 \cdot (2x-3)^2$

b) $f(x) = 2 \cdot \sqrt{x^2+3} \Rightarrow f'(x) = 2 \cdot \frac{2x}{2 \cdot \sqrt{x^2+3}} = \frac{2x}{\sqrt{x^2+3}}$

c) $f(x) = (x^2+3x) \cdot (x^2-1) \Rightarrow f(x) = x^4+3x^3-x^2-3x \Rightarrow f'(x) = 4x^3+9x^2-2x-3$

d) $f(x) = \frac{2x}{x^2+1} \Rightarrow f'(x) = \frac{2 \cdot (x^2+1) - 2x \cdot 2x}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2} = \frac{2 \cdot (1-x^2)}{(x^2+1)^2}$

e) $f(x) = \frac{1+x^2}{2x} \Rightarrow f'(x) = \frac{2x \cdot 2x - (1+x^2) \cdot 2}{4x^2} = \frac{2x^2-2}{4x^2} = \frac{x^2-1}{2x^2} = \frac{1}{2} - \frac{1}{2x^2}$

f) $f(x) = \frac{\sqrt{2x+1}}{x^2+1} \Rightarrow f'(x) = \frac{(x^2+1) \cdot \frac{2}{2 \cdot \sqrt{2x+1}} - \sqrt{2x+1} \cdot 2x}{(x^2+1)^2} = \frac{(x^2+1) - (2x+1) \cdot 2x}{(x^2+1)^2 \cdot \sqrt{2x+1}} =$
 $\frac{x^2+1-4x^2-2x}{(x^2+1)^2 \cdot \sqrt{2x+1}} = \frac{1-2x-3x^2}{(x^2+1)^2 \cdot \sqrt{2x+1}}$

g) $f(x) = \frac{3x-1}{\sqrt{x^2+1}} \Rightarrow f'(x) = \frac{3 \cdot \sqrt{x^2+1} - (3x-1) \cdot \frac{2x}{2 \cdot \sqrt{x^2+1}}}{x^2+1} = \frac{3 \cdot (x^2+1) - (3x-1) \cdot x}{(x^2+1) \cdot \sqrt{x^2+1}} =$
 $\frac{3x^2+3-3x^2+x}{(x^2+1) \cdot \sqrt{x^2+1}} = \frac{3+x}{(x^2+1) \cdot \sqrt{x^2+1}}$

h) $f(x) = \frac{x^2-1}{x^2+1} \Rightarrow f'(x) = \frac{2x \cdot (x^2+1) - (x^2-1) \cdot 2x}{(x^2+1)^2} = \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$

i) $f(x) = -x + x \cdot \ln(x) \Rightarrow f'(x) = -1 + 1 \cdot \ln(x) + x \cdot \frac{1}{x} = -1 + \ln(x) + 1 = \ln(x)$

j) $f(x) = 2x \cdot \ln(1+x^2) \Rightarrow f'(x) = 2 \cdot \ln(1+x^2) + 2x \cdot \frac{1 \cdot 2x}{x^2+1} = \frac{4x^2}{x^2+1} + 2 \cdot \ln(1+x^2)$

k) $f(x) = \ln\left(\frac{2x}{x^2+1}\right) \Rightarrow f'(x) = \frac{x^2+1}{2x} \cdot \frac{(x^2+1) \cdot 2 - 2x \cdot 2x}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{2x \cdot (x^2+1)} = \frac{2-2x^2}{2x \cdot (x^2+1)} = \frac{1-x^2}{x \cdot (x^2+1)}$

l) $f(x) = \ln(\sqrt{x^2+1}) \Rightarrow f'(x) = \frac{1}{\sqrt{x^2+1}} \cdot \frac{2x}{2 \cdot \sqrt{x^2+1}} = \frac{x}{x^2+1}$

m) $f(x) = \ln(x \cdot \sqrt{x^2+1}) \Rightarrow f'(x) = \frac{1}{x \cdot \sqrt{x^2+1}} \cdot (1 \cdot \sqrt{x^2+1} + x \cdot \frac{2x}{2 \cdot \sqrt{x^2+1}}) =$
 $= \frac{1}{x \cdot \sqrt{x^2+1}} \cdot \frac{(x^2+1) + x^2}{\sqrt{x^2+1}} = \frac{2x^2+1}{x \cdot (x^2+1)}$

n) $f(x) = 2x \cdot e^{x+1} \Rightarrow f'(x) = 2 \cdot e^{x+1} + 2x \cdot e^{x+1} = (2+2x) \cdot e^{x+1} = 2 \cdot (x+1) \cdot e^{x+1}$

o) $f(x) = (x^2-x) \cdot e^{2x} \Rightarrow f'(x) = (2x-1) \cdot e^{2x} + (x^2-x) \cdot e^{2x} \cdot 2 = (2x-1+2x^2-2x) \cdot e^{2x} = (2x^2-1) \cdot e^{2x}$

p) $f(x) = \frac{2-e^x}{1+e^x} \Rightarrow f'(x) = \frac{(1+e^x) \cdot (-e^x) - (2-e^x) \cdot e^x}{(1+e^x)^2} = \frac{-e^x - e^{2x} - 2e^x + e^{2x}}{(1+e^x)^2} = \frac{-3 \cdot e^x}{(1+e^x)^2}$

q) $f(x) = \frac{e^{2x+1}}{e^{2x}+1} \Rightarrow f'(x) = \frac{(e^{2x}+1) \cdot e^{2x+1} \cdot 2 - e^{2x+1} \cdot e^{2x} \cdot 2}{(e^{2x}+1)^2} = \frac{2e^{4x+1} + 2e^{2x+1} - 2e^{4x+1}}{(e^{2x}+1)^2} = \frac{2e^{2x+1}}{(e^{2x}+1)}$



2.

$$\text{a) } \int_1^3 \frac{2x}{\sqrt{x^2+3}} dx \stackrel{\text{ib)}}{=} \left[2 \cdot \sqrt{x^2+3} \right]_1^3 = 2 \cdot \sqrt{9+3} - 2 \cdot \sqrt{1+3} = 4 \cdot \sqrt{3} - 4 = 4 \cdot (\sqrt{3} - 1) \approx 2,93$$

$$\text{b) } \int_{-2}^{-1} \frac{x^2-1}{2x^2} dx \stackrel{\text{ie)}}{=} \left[\frac{1+x^2}{2x} \right]_{-2}^{-1} = \frac{1+1}{-2} - \frac{1+4}{-4} = -1 + \frac{5}{4} = \frac{1}{4} \quad \text{oder}$$

$$\int_{-2}^{-1} \frac{x^2-1}{2x^2} dx = \int_{-2}^{-1} \frac{1}{2} - \frac{1}{2x^2} dx = \left[\frac{x}{2} + \frac{1}{2x} \right]_{-2}^{-1} = \frac{-1}{2} + \frac{1}{-2} - \left(\frac{-2}{2} + \frac{1}{-4} \right) = -1 + \frac{5}{4} = \frac{1}{4}$$

$$\text{c) } \int_0^1 \frac{3 \cdot e^x}{(1+e^x)^2} dx \stackrel{\text{ip)}}{=} - \left[\frac{2-e^x}{1+e^x} \right]_0^1 = - \left(\frac{2-e}{1+e} \right) + \frac{2-1}{1+1} = \frac{e-2}{e+1} + \frac{1}{2} \approx 0,693$$

$$\text{d) } \int_1^e \ln(x) dx \stackrel{\text{li)}}{=} \left[-x + x \cdot \ln(x) \right]_1^e = -e + e \cdot \ln e - (-1 + 1 \cdot \ln(1)) = 0 - (-1 + 1 \cdot 0) = 1$$

$$\text{e) } \int_0^1 (2x^2-1) \cdot e^{2x} dx \stackrel{\text{io)}}{=} \left[(x^2-x) \cdot e^{2x} \right]_0^1 = (1-1) \cdot e^2 - (0-0) \cdot e^0 = 0$$

