

# Mathematik \* Jahrgangsstufe 9 \* Potenzen mit rationalen Exponenten

1. Mache den Nenner rational!

Beispiel 1:  $\frac{1}{\sqrt[3]{4x}} = \frac{1 \cdot \sqrt[3]{2x^2}}{\sqrt[3]{4x} \cdot \sqrt[3]{2x^2}} = \frac{\sqrt[3]{2x^2}}{\sqrt[3]{2^3 x^3}} = \frac{\sqrt[3]{2x^2}}{2x}$

Beispiel 2:  $\frac{6}{\sqrt[3]{4x^2} \sqrt{x}} = \frac{6}{(2^2 \cdot x^2 \cdot x^{\frac{1}{2}})^{\frac{1}{3}}} = \frac{6}{(2^{\frac{5}{3}} \cdot x^{\frac{5}{3}})^{\frac{1}{3}}} = \frac{6 \cdot (2^{\frac{1}{3}} \cdot x^{\frac{1}{6}})}{(2^{\frac{5}{3}} \cdot x^{\frac{5}{6}}) \cdot (2^{\frac{1}{3}} \cdot x^{\frac{1}{6}})} = \frac{6 \cdot \sqrt[3]{2} \cdot \sqrt[6]{x}}{2x} = \frac{3 \cdot \sqrt[3]{2} \cdot \sqrt[6]{x}}{x} (= \frac{3 \cdot \sqrt[6]{4x}}{x})$

a)  $\frac{2}{\sqrt[4]{8x^3}}$

b)  $\frac{6}{\sqrt[3]{12xy^2}}$

c)  $\frac{15}{\sqrt[5]{81x^3}}$

d)  $\frac{8}{\sqrt[3]{4x} \cdot \sqrt{x}}$

e)  $\frac{9}{\sqrt[4]{27x^3} \cdot \sqrt[3]{3x}}$

f)  $\frac{5}{\sqrt[3]{x} \cdot \sqrt{xy}}$



2. Vereinfache so weit wie möglich und schreibe als Wurzelterm!

Nenner sind rational zu machen!

Beispiel 1:  $\sqrt[4]{8x^3} \cdot \sqrt[3]{4x} = (2^3 \cdot x^3)^{\frac{1}{4}} \cdot (2^2 \cdot x)^{\frac{1}{3}} = 2^{\frac{3}{4}} \cdot x^{\frac{3}{4}} \cdot 2^{\frac{2}{3}} \cdot x^{\frac{1}{3}} = 2^{\frac{9}{12} + \frac{8}{12}} \cdot x^{\frac{9}{12} + \frac{4}{12}} = 2^{\frac{17}{12}} \cdot x^{\frac{13}{12}} = 2^{1 + \frac{5}{12}} \cdot x^{1 + \frac{1}{12}} = 2x \cdot \sqrt[12]{2^5 \cdot x} = 2x \cdot \sqrt[12]{32x}$

Beispiel 2:  $\sqrt[4]{2x^3} \cdot \sqrt[3]{4x^2} = ((2 \cdot x^3 \cdot (2^2 \cdot x^2)^{\frac{1}{3}})^{\frac{1}{4}} = ((2 \cdot x^3 \cdot 2^{\frac{2}{3}} \cdot x^{\frac{2}{3}})^{\frac{1}{4}} = (2^{\frac{8}{3}} \cdot x^{\frac{11}{3}})^{\frac{1}{4}} = 2^{\frac{8}{12}} \cdot x^{\frac{11}{12}} = \sqrt[12]{2^8 \cdot x^{11}} (= \sqrt[3]{4} \cdot \sqrt[12]{x^{11}})$



a)  $\sqrt[4]{8x^3} \cdot \sqrt{12x}$

b)  $\sqrt[3]{12xy^2} \cdot \sqrt[4]{6xy}$

c)  $\frac{\sqrt[4]{x^2} \cdot \sqrt{x}}{\sqrt[8]{x}}$

d)  $\frac{\sqrt{2} \cdot \sqrt[3]{4} \cdot \sqrt[3]{4} \cdot \sqrt{2}}{\sqrt[3]{4} \cdot \sqrt[4]{8}}$

e)  $\frac{\sqrt[4]{x^2} \cdot \sqrt[6]{x}}{\sqrt[3]{x}}$

f)  $\frac{\sqrt[4]{x^3} \cdot \sqrt[3]{x^2}}{\sqrt[6]{x} \cdot \sqrt[4]{x^3} \cdot \sqrt[8]{x}}$

g)  $\frac{\sqrt[3]{6x} \cdot \sqrt{3x}}{\sqrt[4]{4} \cdot \sqrt[6]{4}}$

h)  $\frac{\sqrt{x} \cdot \sqrt[3]{x^2} \cdot \sqrt[4]{x}}{\sqrt[6]{x^4} \cdot \sqrt{x}}$

3. Löse die Gleichung!

Beispiel 1:  $4 \cdot (x^5 - 3) = x^5 - 204 \Leftrightarrow 4x^5 - 12 = x^5 - 204 \Leftrightarrow 3x^5 = -192 \Leftrightarrow$

$x^5 = -64 \Leftrightarrow x = -\sqrt[5]{64} = -\sqrt[5]{2^6} = -2 \cdot \sqrt[5]{2}$

Beispiel 2:  $4 \cdot x^4 - 3 = 969 \Leftrightarrow 4 \cdot x^4 = 972 \Leftrightarrow x^4 = 243 \Leftrightarrow x_{1/2} = \pm \sqrt[4]{3 \cdot 3^4} = \pm 3 \cdot \sqrt[4]{3}$

a)  $2 \cdot x^3 - 45 = 67$

b)  $\frac{1}{16} \cdot x^5 + 87 = 65$

c)  $3 \cdot x^4 - 5 = 67$

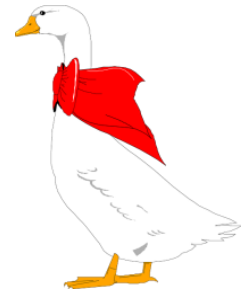
d)  $7 \cdot x^4 + 56 = 7$

e)  $\frac{2x^4 - 8}{6} = 7$

f)  $\frac{0,1 \cdot x^3 + 5}{5} = 6$

g)  $\frac{159}{x^3 + 5} = -3$

h)  $\frac{95}{0,5 \cdot x^4 + 3} = 5$



1.a) 
$$\frac{2}{\sqrt[4]{8x^3}} = \frac{2 \cdot \sqrt[4]{2x}}{\sqrt[4]{2^3 x^3 \cdot \sqrt[4]{2x}}} = \frac{2 \cdot \sqrt[4]{2x}}{2x} = \frac{\sqrt[4]{2x}}{x}$$

b) 
$$\frac{6}{\sqrt[3]{12xy^2}} = \frac{6 \cdot \sqrt[3]{2 \cdot 3^2 \cdot x^2 y}}{\sqrt[3]{2^2 \cdot 3 \cdot xy^2 \cdot \sqrt[3]{2 \cdot 3^2 \cdot x^2 y}}} = \frac{6 \cdot \sqrt[3]{2 \cdot 3^2 \cdot x^2 y}}{2 \cdot 3 \cdot x \cdot y} = \frac{\sqrt[3]{18 \cdot x^2 y}}{xy}$$

c) 
$$\frac{15}{\sqrt[5]{81x^3}} = \frac{15 \cdot \sqrt[5]{3 \cdot x^2}}{\sqrt[5]{3^4 \cdot x^3 \cdot \sqrt[5]{3 \cdot x^2}}} = \frac{15 \cdot \sqrt[5]{3 \cdot x^2}}{3x} = \frac{5 \cdot \sqrt[5]{3 \cdot x^2}}{x}$$

d) 
$$\frac{8}{\sqrt[3]{4x \cdot \sqrt{x}}} = \frac{8}{(2^2 \cdot x \cdot x^{\frac{1}{2}})^{\frac{1}{3}}} = \frac{8}{2^{\frac{2}{3}} \cdot x^{\frac{3}{2 \cdot 3}}} = \frac{8 \cdot 2^{\frac{1}{3}} \cdot x^{\frac{1}{2}}}{2^{\frac{2}{3}} \cdot x^{\frac{1}{2}} \cdot 2^{\frac{1}{3}} \cdot x^{\frac{1}{2}}} = \frac{8 \cdot 2^{\frac{1}{3}} \cdot x^{\frac{1}{2}}}{2x} = \frac{8 \cdot \sqrt[3]{2} \cdot \sqrt{x}}{2x} \quad (= \frac{8 \cdot \sqrt[6]{4x^3}}{2x})$$

e) 
$$\frac{9}{\sqrt[4]{27x^3 \cdot \sqrt{3x}}} = \frac{9}{(3^3 \cdot x^3 \cdot 3^{\frac{1}{2}} \cdot x^{\frac{1}{2}})^{\frac{1}{4}}} = \frac{9}{3^{\frac{7}{8}} \cdot x^{\frac{7}{8}}} = \frac{9 \cdot 3^{\frac{1}{8}} \cdot x^{\frac{1}{8}}}{3^{\frac{7}{8}} \cdot x^{\frac{7}{8}} \cdot 3^{\frac{1}{8}} \cdot x^{\frac{1}{8}}} = \frac{9 \cdot 3^{\frac{1}{8}} \cdot x^{\frac{1}{8}}}{3x} = \frac{9 \cdot \sqrt[8]{3x}}{3x}$$

f) 
$$\frac{5}{\sqrt[3]{x \cdot \sqrt{xy}}} = \frac{5}{(x \cdot x^{\frac{1}{2}} \cdot y^{\frac{1}{2}})^{\frac{1}{3}}} = \frac{5}{x^{\frac{3}{2 \cdot 3}} \cdot y^{\frac{1}{2 \cdot 3}}} = \frac{5 \cdot x^{\frac{1}{2}} \cdot y^{\frac{5}{6}}}{x^{\frac{1}{2}} \cdot y^{\frac{1}{6}} \cdot x^{\frac{1}{2}} \cdot y^{\frac{5}{6}}} = \frac{5 \cdot \sqrt{x} \cdot \sqrt[6]{y^5}}{xy} \quad (= \frac{5 \cdot \sqrt[6]{x^3 y^5}}{xy})$$

2.a) 
$$\sqrt[4]{8x^3} \cdot \sqrt{12x} = (2^3 \cdot x^3)^{\frac{1}{4}} \cdot (2^2 \cdot 3 \cdot x)^{\frac{1}{2}} = 2^{\frac{3}{4}} \cdot x^{\frac{3}{4}} \cdot 2 \cdot 3^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = 2 \cdot 2^{\frac{3}{4}} \cdot x^{1+\frac{1}{4}} \cdot 3^{\frac{1}{2}} = 2x \cdot \sqrt[4]{9x}$$

b) 
$$\sqrt[3]{12xy^2} \cdot \sqrt{6xy} = (2^2 \cdot 3 \cdot x \cdot y^2 \cdot (2 \cdot 3 \cdot x \cdot y)^{\frac{1}{2}})^{\frac{1}{3}} = (2^{\frac{5}{2}} \cdot 3^{\frac{3}{2}} \cdot x^{\frac{3}{2}} \cdot y^{\frac{5}{2}})^{\frac{1}{3}} = 2^{\frac{5}{6}} \cdot 3^{\frac{3}{6}} \cdot x^{\frac{3}{6}} \cdot y^{\frac{5}{6}} = \sqrt[6]{2^5 \cdot 3^3 \cdot x^3 \cdot y^5} \quad (= \sqrt[6]{2^5 \cdot y^5} \cdot \sqrt{3x})$$

c) 
$$\frac{\sqrt[4]{x^2 \cdot \sqrt{x}}}{\sqrt[8]{x}} = \frac{(x^{2+\frac{1}{2}})^{\frac{1}{4}}}{x^{\frac{1}{8}}} = x^{\frac{5-\frac{1}{8}}{8}} = x^{\frac{4}{8}} = x^{\frac{1}{2}} = \sqrt{x}$$

d) 
$$\frac{\sqrt{2 \cdot \sqrt[3]{4}} \cdot \sqrt[3]{4 \cdot \sqrt{2}}}{\sqrt[3]{4 \cdot \sqrt[4]{8}}} = \frac{(2 \cdot 2^{\frac{2}{3}})^{\frac{1}{2}} \cdot (2^2 \cdot 2^{\frac{1}{2}})^{\frac{1}{3}}}{(2^2 \cdot 2^{\frac{3}{4}})^{\frac{1}{3}}} = \frac{2^{\frac{1}{3}} \cdot 2^{\frac{5}{6}}}{2^{\frac{11-1}{4 \cdot 3}}} = 2^{\frac{4}{12} + \frac{10}{12} - \frac{11}{12}} = 2^{\frac{3}{12}} = 2^{\frac{1}{4}} = \sqrt[4]{2}$$

e) 
$$\frac{\sqrt[4]{x^2 \cdot \sqrt[6]{x}}}{\sqrt[3]{x}} = \frac{x^{\frac{2}{4}} \cdot x^{\frac{1}{6}}}{x^{\frac{1}{3}}} = x^{\frac{6}{12} + \frac{2}{12} - \frac{4}{12}} = x^{\frac{4}{12}} = x^{\frac{1}{3}} = \sqrt[3]{x}$$

f) 
$$\frac{\sqrt[4]{x^3 \cdot \sqrt[3]{x^2}}}{\sqrt[6]{x \cdot \sqrt[4]{x^3}} \cdot \sqrt{x}} = \frac{(x^{\frac{1}{3}} \cdot x^{\frac{2}{3}})^{\frac{1}{4}}}{(x \cdot x^{\frac{3}{4}})^{\frac{1}{6}} \cdot x^{\frac{1}{2}}} = \frac{x^{\frac{31}{34}}}{x^{\frac{7}{4 \cdot 6}} \cdot x^{\frac{1}{8}}} = x^{\frac{1}{4} - \frac{7}{24} - \frac{1}{8}} = x^{\frac{6}{24} - \frac{7}{24} - \frac{3}{24}} = x^{-\frac{4}{24}} = x^{-1+\frac{5}{6}} = \frac{\sqrt[6]{x^5}}{x}$$

g) 
$$\frac{\sqrt[3]{6x \cdot \sqrt{3x}}}{\sqrt[4]{4 \cdot \sqrt[6]{4}}} = \frac{(2 \cdot 3 \cdot 3^{\frac{1}{2}} \cdot x^{\frac{1}{2}})^{\frac{1}{3}}}{2^{\frac{2}{4}} \cdot 2^{\frac{2}{6}}} = \frac{2^{\frac{1}{3}} \cdot 3^{\frac{31}{23}} \cdot x^{\frac{1}{6}}}{2^{\frac{1}{6}}} = 2^{\frac{2}{6} - \frac{1}{6}} \cdot 3^{\frac{3}{6}} \cdot x^{\frac{1}{6}} = \sqrt[6]{2 \cdot 3^3 \cdot x} \quad (= \sqrt[6]{2x} \cdot \sqrt{3})$$

h) 
$$\frac{\sqrt{x \cdot \sqrt[3]{x^2 \cdot \sqrt[4]{x}}}}{\sqrt[6]{x^4 \cdot \sqrt{x}}} = \frac{(x \cdot (x^2 \cdot x^{\frac{1}{4}})^{\frac{1}{3}})^{\frac{1}{2}}}{(x^4 \cdot x^{\frac{1}{2}})^{\frac{1}{6}}} = \frac{(x \cdot x^{\frac{91}{43}})^{\frac{1}{2}}}{(x^{\frac{9}{2}})^{\frac{1}{6}}} = x^{(1+\frac{3}{4}) \cdot \frac{1}{2} - \frac{91}{2 \cdot 6}} = x^{\frac{7}{8} - \frac{3}{4}} = x^{\frac{1}{8}} = \sqrt[8]{x}$$

3. a)  $2 \cdot x^3 - 45 = 67 \Leftrightarrow 2 \cdot x^3 = 112 \Leftrightarrow x^3 = 56 \Leftrightarrow x = \sqrt[3]{56} = \sqrt[3]{8 \cdot 7} = 2 \cdot \sqrt[3]{7}$
- b)  $\frac{1}{16} \cdot x^5 + 87 = 65 \Leftrightarrow x^5 = 16 \cdot (65 - 87) \Leftrightarrow x^5 = -16 \cdot 22 \Leftrightarrow x = -\sqrt[5]{2^4 \cdot 2 \cdot 11} = -2 \cdot \sqrt[5]{11}$
- c)  $3 \cdot x^4 - 5 = 67 \Leftrightarrow x^4 = \frac{1}{3} \cdot 72 \Leftrightarrow x^4 = 24 \Leftrightarrow x_{1/2} = \pm \sqrt[4]{24}$
- d)  $7 \cdot x^4 + 56 = 7 \Leftrightarrow x^4 = \frac{1}{7} \cdot (7 - 56) \Leftrightarrow x^4 < 0$  also keine Lösung, d.h.  $L = \{ \}$
- e)  $\frac{2x^4 - 8}{6} = 7 \Leftrightarrow 2x^4 = 6 \cdot 7 + 8 \Leftrightarrow x^4 = 25 \Leftrightarrow x_{1/2} = \pm (5^2)^{\frac{1}{4}} = \pm 5^{\frac{1}{2}} = \pm \sqrt{5}$
- f)  $\frac{0,1 \cdot x^3 + 5}{5} = 6 \Leftrightarrow 0,1 \cdot x^3 = 30 - 5 \Leftrightarrow x^3 = 250 \Leftrightarrow x = \sqrt[3]{250} = \sqrt[3]{5^3 \cdot 2} = 5 \cdot \sqrt[3]{2}$
- g)  $\frac{159}{x^3 + 5} = -3 \Leftrightarrow 159 = -3x^3 - 15 \Leftrightarrow 3x^3 = -15 - 159 \Leftrightarrow x^3 = -\frac{174}{3} \Leftrightarrow x = -\sqrt[3]{58}$
- h)  $\frac{95}{0,5 \cdot x^4 + 3} = 5 \Leftrightarrow 95 = 2,5x^4 + 15 \Leftrightarrow \frac{5}{2} \cdot x^4 = 80 \Leftrightarrow x^4 = \frac{160}{5} \Leftrightarrow x^4 = 32 \Leftrightarrow$   
 $x_{1/2} = \pm \sqrt[4]{32} = \pm 2 \cdot \sqrt[4]{2}$

