

# Mathematik \* Jahrgangsstufe 9 \* Potenzen mit rationalen Exponenten

Beachte folgende Potenzgesetze:

$$a^x \cdot a^y = a^{x+y} \quad ; \quad \frac{a^x}{a^y} = a^x : a^y = a^{x-y} \quad ; \quad (a^x)^y = a^{x \cdot y} \quad \text{und natürlich} \quad a^x \cdot b^x = (a \cdot b)^x$$

$$a^{-x} = \frac{1}{a^x} \quad \text{und} \quad \sqrt[n]{a} = a^{\frac{1}{n}} \quad \text{und} \quad \sqrt[n]{a^m} = a^{\frac{m}{n}}$$



1. Schreibe als Potenz mit rationalem Exponent!

Beispiel 1 :  $\sqrt{2a \cdot \sqrt[3]{4a}} = (2a \cdot (2^2 \cdot a)^{\frac{1}{3}})^{\frac{1}{2}} = (2 \cdot a \cdot 2^{\frac{2}{3}} \cdot a^{\frac{1}{3}})^{\frac{1}{2}} = (2^{\frac{5}{3}} \cdot a^{\frac{4}{3}})^{\frac{1}{2}} = 2^{\frac{5}{6}} \cdot a^{\frac{4}{6}} = 2^{\frac{5}{6}} \cdot a^{\frac{2}{3}}$

Beispiel 2 :  $(\sqrt[5]{4x^3})^2 = ((2^2 \cdot x^3)^{\frac{1}{5}})^2 = (2^2 \cdot x^3)^{\frac{2}{5}} = 2^{\frac{4}{5}} \cdot x^{\frac{6}{5}}$

a)  $\sqrt[3]{4x^2 \cdot \sqrt{2x}}$       b)  $(\sqrt[3]{18z^2})^2$       c)  $\sqrt[3]{\sqrt[3]{96y^2}}$   
 d)  $(\sqrt[6]{48 \cdot y^4})^4$       e)  $(\sqrt[3]{50a^3 \cdot \sqrt{5}})^2$       f)  $(\sqrt[4]{27x^3 \sqrt{3x}})^3$



2. Vereinfache so weit wie möglich und schreibe als Wurzelterm! Nenner sind rational zu machen!

Beispiel 1 :  $\sqrt[4]{8x^3} \cdot \sqrt[4]{4x} = (8x^3 \cdot 4x)^{\frac{1}{4}} = (2^5 \cdot x^4)^{\frac{1}{4}} = 2 \cdot x \cdot 2^{\frac{1}{4}} = 2x \cdot \sqrt[4]{2}$

Beispiel 2 :  $\frac{\sqrt[4]{8x}}{\sqrt{2x}} = \frac{\sqrt[4]{8x}}{\sqrt[4]{4x^2}} = \sqrt[4]{\frac{8x}{4x^2}} = \sqrt[4]{\frac{2}{x}} = \sqrt[4]{\frac{2 \cdot x^3}{x \cdot x^3}} = \frac{\sqrt[4]{2x^3}}{x}$

a)  $\sqrt[3]{4x^2} \cdot \sqrt[3]{6x}$       b)  $\sqrt[3]{10x^2} \cdot \sqrt[3]{20x}$       c)  $\sqrt[4]{18y^3} : \sqrt{y \cdot \sqrt{4y}}$   
 d)  $\sqrt[6]{24z^4} \cdot \sqrt[3]{18z^2}$       e)  $\sqrt[6]{16y^5} \cdot \sqrt[3]{4y^2}$       f)  $\sqrt{6x} \cdot \sqrt[4]{27x^3} : \sqrt[4]{3x}$

3. Schreibe nur mit einer Wurzel! Nenner sind rational zu machen!

Beispiel 1 :  $\sqrt[4]{24} \cdot \sqrt[3]{\sqrt{12}} = 24^{\frac{1}{4}} \cdot (12^{\frac{1}{2}})^{\frac{1}{3}} = 24^{\frac{1}{4}} \cdot (12^{\frac{1}{2 \cdot 3}})^{\frac{1}{3}} = 24^{\frac{1}{4}} \cdot 12^{\frac{2}{12}} = (24^3 \cdot 12^2)^{\frac{1}{12}} =$   
 $((2^3 \cdot 3)^3 \cdot (2^2 \cdot 3)^2)^{\frac{1}{12}} = (2^9 \cdot 3^3 \cdot 2^4 \cdot 3^2)^{\frac{1}{12}} = (2^{13} \cdot 3^5)^{\frac{1}{12}} = 2^{\frac{13}{12}} \cdot 3^{\frac{5}{12}} = 2 \cdot 2^{\frac{1}{12}} \cdot 3^{\frac{5}{12}} = 2 \cdot \sqrt[12]{2 \cdot 3^5}$

Beispiel 2 :  $\sqrt[3]{5} : \sqrt{2} = 5^{\frac{1}{3 \cdot 4}} : 2^{\frac{1 \cdot 3}{4 \cdot 3}} = 5^{\frac{1}{12}} : 2^{\frac{3}{12}} = \left(\frac{5^4}{2^3}\right)^{\frac{1}{12}} = \left(\frac{5^4 \cdot 2^9}{2^3 \cdot 2^9}\right)^{\frac{1}{12}} = \left(\frac{5^4 \cdot 2^9}{2^{12}}\right)^{\frac{1}{12}} =$   
 $\frac{(5^4 \cdot 2^9)^{\frac{1}{12}}}{2} = \frac{\sqrt[12]{5^4 \cdot 2^9}}{2}$

a)  $\sqrt{8} \cdot \sqrt[3]{4}$       b)  $\sqrt[3]{18} : \sqrt{6}$       c)  $\sqrt[4]{50} \cdot \sqrt[3]{40}$   
 d)  $\sqrt{6} : \sqrt[3]{12}$       e)  $\sqrt[3]{4} \cdot \sqrt[4]{24} : \sqrt{3}$       f)  $\sqrt{15} \cdot \sqrt[3]{16}$   
 g)  $\sqrt{2 \cdot \sqrt[3]{6}}$       h)  $\sqrt[4]{\sqrt{24}} \cdot \sqrt[8]{96}$       k)  $\sqrt[3]{\frac{\sqrt[4]{18}}{2 \cdot \sqrt{6}}}$



Mathematik \* Jahrgangsstufe 9 \* Potenzen mit rationalen Exponenten \* Lösungen

1. a)  $\sqrt[3]{4x^2 \cdot \sqrt{2x}} = \left(4x^2 \cdot (2x)^{\frac{1}{2}}\right)^{\frac{1}{3}} = \left(2^2 x^2 \cdot 2^{\frac{1}{2}} x^{\frac{1}{2}}\right)^{\frac{1}{3}} = \left(2^{\frac{5}{2}} x^{\frac{5}{2}}\right)^{\frac{1}{3}} = 2^{\frac{5}{6}} x^{\frac{5}{6}}$

b)  $\left(\sqrt[3]{18z^2}\right)^2 = \left(\left(2 \cdot 3^2 z^2\right)^{\frac{1}{3}}\right)^2 = \left(2 \cdot 3^2 z^2\right)^{\frac{2}{3}} = 2^{\frac{2}{3}} \cdot 3^{\frac{4}{3}} z^{\frac{4}{3}} (= 3z \cdot 2^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} z^{\frac{1}{3}} = 3z \cdot \sqrt[3]{12z})$

c)  $\sqrt[3]{\sqrt{96y^2}} = \left(3 \cdot 2^5 \cdot y^2\right)^{\frac{1}{3} \cdot \frac{1}{2}} = 3^{\frac{1}{6}} \cdot 2^{\frac{5}{6}} \cdot y^{\frac{1}{3}}$

d)  $\left(\sqrt[6]{48 \cdot y^4}\right)^4 = \left(2^4 \cdot 3 \cdot y^4\right)^{\frac{1}{6} \cdot 4} = \left(2^4 \cdot 3 \cdot y^4\right)^{\frac{2}{3}} = 2^{\frac{8}{3}} \cdot 3^{\frac{2}{3}} \cdot y^{\frac{8}{3}} (= 4 \cdot y^2 \cdot \sqrt[3]{4 \cdot 9 \cdot y^2} = 4 \cdot y^2 \cdot \sqrt[3]{36 \cdot y^2})$

e)  $\left(\sqrt[3]{50a^3 \cdot \sqrt{5}}\right)^2 = \left(\left(5^2 \cdot 2a^3\right) \cdot 5^{\frac{1}{2}}\right)^{\frac{1}{3} \cdot 2} = \left(5^{\frac{5}{2}} \cdot 2^{\frac{1}{2}} a^{\frac{3}{2}}\right)^{\frac{2}{3}} = 5^{\frac{5}{3}} \cdot 2^{\frac{1}{3}} a^1 = 5^{1+\frac{2}{3}} \cdot 2^{\frac{1}{3}} a^1 = 5a \cdot \sqrt[3]{50}$

f)  $\left(\sqrt[4]{27x^3 \sqrt{3x}}\right)^3 = \left(\left(3^3 x^3 \cdot (3x)^{\frac{1}{2}}\right)^{\frac{1}{4}}\right)^3 = \left(\left(3^{\frac{7}{2}} x^{\frac{7}{2}}\right)^{\frac{1}{4}}\right)^3 = \left(3^{\frac{7}{2}} x^{\frac{7}{2}}\right)^{\frac{3}{4}} = 3^{\frac{21}{8}} x^{\frac{21}{8}}$   
 $(= 9x^2 \cdot 3^{\frac{5}{8}} x^{\frac{5}{8}} = 9x^2 \cdot \sqrt[8]{32x^5})$



2. a)  $\sqrt[3]{4x^2} \cdot \sqrt[3]{6x} = \left(2^2 \cdot x^2\right)^{\frac{1}{3}} \cdot \left(2 \cdot 3 \cdot x\right)^{\frac{1}{3}} = \left(2^3 \cdot 3 \cdot x^3\right)^{\frac{1}{3}} = 2^1 \cdot 3^{\frac{1}{3}} \cdot x^1 = 2x \cdot \sqrt[3]{3}$

b)  $\sqrt[3]{10x^2} \cdot \sqrt[3]{20x} = \left(2 \cdot 5 \cdot x^2 \cdot 2^2 \cdot 5 \cdot x\right)^{\frac{1}{3}} = \left(2^3 \cdot 5^2 \cdot x^3\right)^{\frac{1}{3}} = 2 \cdot x \cdot 5^{\frac{2}{3}} = 2x \cdot \sqrt[3]{25}$

c)  $\sqrt[4]{18y^3} : \sqrt{y \cdot \sqrt{4y}} = \left(2 \cdot 3^2 \cdot y^3\right)^{\frac{1}{4}} \cdot \left(y \cdot (2^2 \cdot y)^{\frac{1}{2}}\right)^{-\frac{1}{2}} = 2^{\frac{1}{4}} \cdot 3^{\frac{3}{4}} \cdot y^{\frac{3}{4}} \cdot \left(2 \cdot y^{\frac{3}{2}}\right)^{-\frac{1}{2}} =$   
 $2^{\frac{1}{4}} \cdot 3^{\frac{3}{4}} \cdot y^{\frac{3}{4}} \cdot 2^{-\frac{1}{2}} \cdot y^{-\frac{3}{4}} = 2^{-\frac{1}{4}} \cdot 3^{\frac{3}{4}} \cdot y^0 = 2^{-\frac{1}{4}} \cdot 3^{\frac{3}{4}} = \frac{\sqrt[4]{8 \cdot 9}}{2} = \frac{\sqrt[4]{72}}{2}$

d)  $\sqrt[6]{24z^4} \cdot \sqrt[3]{18z^2} = \left(2^3 \cdot 3 \cdot z^4\right)^{\frac{1}{6}} \cdot \left(2 \cdot 3^2 \cdot z^2\right)^{\frac{1}{3}} = \left(2^3 \cdot 3 \cdot z^4\right)^{\frac{1}{6}} \cdot \left(2^2 \cdot 3^4 \cdot z^4\right)^{\frac{1}{6}} =$   
 $\left(2^{3+2} \cdot 3^{1+4} \cdot z^{4+4}\right)^{\frac{1}{6}} = 2^{\frac{5}{6}} \cdot 3^{\frac{5}{6}} \cdot z^{\frac{8}{6}} = 2^{\frac{5}{6}} \cdot 3^{\frac{5}{6}} \cdot z^{\frac{4}{3}} (= z \cdot \sqrt[6]{6^5 \cdot z^2})$

e)  $\sqrt[6]{16y^5} \cdot \sqrt[3]{4y^2} = \left(2^4 \cdot y^5\right)^{\frac{1}{6}} \cdot \left(2^2 \cdot y^2\right)^{\frac{1}{3}} = 2^{\frac{4}{6}} \cdot y^{\frac{5}{6}} \cdot 2^{\frac{2}{3}} \cdot y^{\frac{2}{3}} = 2^{\frac{8}{6}} \cdot y^{\frac{9}{6}} = 2y \cdot (4y^3)^{\frac{1}{6}} = 2y \cdot \sqrt[6]{4y^3}$

f)  $\sqrt{6x} \cdot \sqrt[4]{27x^3} : \sqrt[4]{3x} = \left(2 \cdot 3 \cdot x\right)^{\frac{1}{2}} \cdot \left(3^3 \cdot x^3\right)^{\frac{1}{4}} \cdot \left(3 \cdot x\right)^{-\frac{1}{4}} = 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2} + \frac{3}{4} - \frac{1}{4}} \cdot x^{\frac{1}{2} + \frac{3}{4} - \frac{1}{4}} =$   
 $2^{\frac{1}{2}} \cdot 3^1 \cdot x^1 = 3x \cdot \sqrt{2}$



3. a)  $\sqrt{8} \cdot \sqrt[3]{4} = \left(2^3\right)^{\frac{1}{2}} \cdot \left(2^2\right)^{\frac{1}{3}} = 2^{\frac{3}{2}} \cdot 2^{\frac{2}{3}} = 2^{\frac{9}{6} + \frac{4}{6}} = 2^{\frac{13}{6}} = 2^{2+\frac{1}{6}} = 2^2 \cdot 2^{\frac{1}{6}} = 4 \cdot \sqrt[6]{2}$

b)  $\sqrt[3]{18} : \sqrt{6} = \left(2 \cdot 3^2\right)^{\frac{1}{3}} \cdot \left(2 \cdot 3\right)^{-\frac{1}{2}} = 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}} \cdot 2^{-\frac{1}{2}} \cdot 3^{-\frac{1}{2}} = 2^{\frac{1}{3} - \frac{1}{2}} \cdot 3^{\frac{2}{3} - \frac{1}{2}} = 2^{-\frac{1}{6}} \cdot 3^{\frac{1}{6}} =$   
 $2^{-1+\frac{5}{6}} \cdot 3^{\frac{1}{6}} = \frac{1}{2} \cdot 2^{\frac{5}{6}} \cdot 3^{\frac{1}{6}} = \frac{\sqrt[6]{2^5 \cdot 3}}{2} = \frac{\sqrt[6]{32 \cdot 3}}{2} = \frac{\sqrt[6]{96}}{2}$

c)  $\sqrt[4]{50} \cdot \sqrt[3]{40} = \left(2 \cdot 5^2\right)^{\frac{1}{4}} \cdot \left(2^3 \cdot 5\right)^{\frac{1}{3}} = 2^{\frac{1}{4} + \frac{3}{4}} \cdot 5^{\frac{2}{4} + \frac{1}{3}} = 2^{1+\frac{3}{4}} \cdot 5^{\frac{10}{12}} = 2^{1+\frac{3}{4}} \cdot 5^{\frac{10}{12}}$

$$d) \sqrt{6} : \sqrt[3]{12} = (2 \cdot 3)^{\frac{1}{2}} \cdot (2^2 \cdot 3)^{-\frac{1}{3}} = 2^{\frac{1}{2} - \frac{2}{3}} \cdot 3^{\frac{1}{2} - \frac{1}{3}} = 2^{-\frac{1}{6}} \cdot 3^{\frac{1}{6}} = 2^{-1 + \frac{5}{6}} \cdot 3^{\frac{1}{6}} = \frac{\sqrt[6]{2^5 \cdot 3}}{2} = \frac{\sqrt[6]{96}}{2}$$

$$e) \sqrt[3]{4} \cdot \sqrt[4]{24} : \sqrt{3} = (2^2)^{\frac{1}{3}} \cdot (2^3 \cdot 3)^{\frac{1}{4}} \cdot 3^{-\frac{1}{2}} = 2^{\frac{2}{3} + \frac{3}{4}} \cdot 3^{\frac{1}{4} - \frac{1}{2}} = 2^{\frac{17}{12}} \cdot 3^{-\frac{1}{4}} = 2^{1 + \frac{5}{12}} \cdot 3^{-1 + \frac{9}{12}} = \frac{2 \cdot \sqrt[12]{2^5 \cdot 3^9}}{3}$$

$$f) \sqrt{15} \cdot \sqrt[3]{16} = (3 \cdot 5)^{\frac{1}{2}} \cdot (2^4)^{\frac{1}{3}} = 3^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \cdot 2^{\frac{4}{3}} = 3^{\frac{3}{6}} \cdot 5^{\frac{3}{6}} \cdot 2^{1 + \frac{2}{6}} = 2 \cdot (3^3 \cdot 5^3 \cdot 2^2)^{\frac{1}{6}} = 2 \cdot \sqrt[6]{3^3 \cdot 5^3 \cdot 2^2}$$

$$g) \sqrt{2 \cdot \sqrt[3]{6}} = (2 \cdot (2 \cdot 3)^{\frac{1}{3}})^{\frac{1}{2}} = (2^{\frac{4}{3}} \cdot 3^{\frac{1}{3}})^{\frac{1}{2}} = 2^{\frac{4}{6}} \cdot 3^{\frac{1}{6}} = \sqrt[6]{2^4 \cdot 3} = \sqrt[6]{48}$$

$$h) \sqrt[4]{\sqrt{24}} \cdot \sqrt[8]{96} = \left( (2^3 \cdot 3)^{\frac{1}{2}} \right)^{\frac{1}{4}} \cdot (2^5 \cdot 3)^{\frac{1}{8}} = (2^3 \cdot 3)^{\frac{11}{24}} \cdot 2^{\frac{5}{8}} \cdot 3^{\frac{1}{8}} = 2^{\frac{3}{8} + \frac{5}{8}} \cdot 3^{\frac{1}{8} + \frac{1}{8}} = 2^1 \cdot 3^{\frac{1}{4}} = 2\sqrt[4]{3}$$

$$k) \sqrt[3]{\frac{\sqrt[4]{18}}{2 \cdot \sqrt{6}}} = \left( (2 \cdot 3^2)^{\frac{1}{4}} \cdot 2^{-1} \cdot (2 \cdot 3)^{-\frac{1}{2}} \right)^{\frac{1}{3}} = \left( 2^{\frac{1}{4}} \cdot 3^{\frac{2}{4}} \cdot 2^{-1} \cdot 2^{-\frac{1}{2}} \cdot 3^{-\frac{1}{2}} \right)^{\frac{1}{3}} = (2^{\frac{1}{4} - 1 - \frac{1}{2}} \cdot 3^{\frac{2}{4} - \frac{1}{2}})^{\frac{1}{3}} = (2^{\frac{1}{4} - 1 - \frac{1}{2}} \cdot 3^{\frac{2}{4} - \frac{1}{2}})^{\frac{1}{3}} = (2^{\frac{1}{4} - 1 - \frac{1}{2}} \cdot 3^{\frac{2}{4} - \frac{1}{2}})^{\frac{1}{3}} = (2^{-\frac{5}{4}} \cdot 1)^{\frac{1}{3}} = 2^{-\frac{5}{12}} = 2^{-1 + \frac{7}{12}} = \frac{\sqrt[12]{2^7}}{2} = \frac{\sqrt[12]{128}}{2}$$

