

Mathematik * Jahrgangsstufe 9 * Potenzen mit rationalen Exponenten

Definition der allgemeinen Wurzel

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad \text{und} \quad \sqrt[n]{a^m} = a^{\frac{m}{n}} \quad \text{für } a \in \mathbb{R}_0^+, n \in \{2, 3, 4, 5, \dots\}, m \in \mathbb{Z}$$

Potenzgesetze:

$$(I) \quad a^{\frac{m}{n}} \cdot a^{\frac{s}{t}} = a^{\frac{m}{n} + \frac{s}{t}} \quad (II) \quad a^{\frac{m}{n}} : a^{\frac{s}{t}} = a^{\frac{m}{n} - \frac{s}{t}} \quad (III) \quad \left(a^{\frac{m}{n}}\right)^s = a^{\frac{m \cdot s}{n}}$$

$$(IV) \quad a^{\frac{m}{n}} \cdot b^{\frac{m}{n}} = (ab)^{\frac{m}{n}} \quad (V) \quad a^{\frac{m}{n}} : b^{\frac{m}{n}} = (a:b)^{\frac{m}{n}}$$

Aufgaben:

1. Vereinfache und stelle den Term als Potenz mit rationalem Exponenten dar.

a) $\sqrt[4]{144}$ b) $\sqrt[6]{625}$ c) $\sqrt[9]{125}$ d) $\sqrt[10]{32}$

2. Vereinfache und stelle den Term als Wurzel mit möglichst kleinem Wurzelexponenten dar.

a) $8^{\frac{1}{6}}$ b) $25^{\frac{1}{8}}$ c) $27^{\frac{1}{9}}$ d) $16^{\frac{1}{12}}$
e) $81^{\frac{1}{10}}$ f) $81^{\frac{1}{20}}$ g) $216^{\frac{1}{6}}$ h) $32^{\frac{1}{15}}$

3. Vereinfache durch teilweises Radizieren und schreibe den Term als Wurzel.

a) $\sqrt[4]{512}$ b) $81^{\frac{1}{3}}$ c) $\sqrt[3]{250}$ d) $144^{\frac{3}{5}}$
e) $\sqrt[3]{49^2}$ f) $24^{\frac{2}{3}}$ g) $\sqrt[5]{72^2}$ h) $8^{\frac{3}{4}}$

4. Mache den Nenner rational und schreibe den Term als Wurzel.

a) $\sqrt[4]{\frac{1}{8}}$ b) $6,75^{\frac{1}{3}}$ c) $\frac{2}{\sqrt[3]{25}}$ d) $0,25^{\frac{3}{5}}$
e) $\frac{2}{\sqrt[4]{9}}$ f) $49^{-\frac{1}{3}}$ g) $\frac{6}{\sqrt[6]{27}}$ h) $8^{-\frac{3}{4}}$



5. Bestimme die Lösungsmenge und gib die Lösung als möglichst weit vereinfachten Wurzelterm an.

a) $2 \cdot x^3 = 32$ b) $0,2 \cdot x^3 = -32$ c) $2 - 3x^4 = 5$
d) $2 \cdot x^3 - 4 = 5$ e) $0,25 \cdot x^6 + 4 = 4^{1,5}$ f) $0,5 \cdot (x-1)^3 - 4 = 12$
g) $8 \cdot x^{-3} = 5$ h) $2 \cdot x^{-5} - 5 = 3^3$ i) $0,5 \cdot x^{-5} + 11 = 3$
j) $2 \cdot x^{\frac{1}{3}} + 4 = 5$ k) $0,5 \cdot x^{\frac{2}{3}} + 3 = 5$ l) $(x-2)^{\frac{1}{4}} - 3 = 5$

6. Vereinfache den Term und stelle ihn in Potenz- und Wurzelschreibweise dar.

a) $3^{0,5} \cdot 9^{0,75}$ b) $4^{\frac{1}{5}} \cdot 2^{\frac{1}{10}} : 8^{\frac{1}{4}}$ c) $\sqrt[4]{27} \cdot \sqrt[5]{9} \cdot \sqrt[20]{27}$
d) $\sqrt[3]{4} : \sqrt[3]{36}$ e) $\sqrt[3]{\frac{4}{5}} : \sqrt[6]{25}$ f) $\sqrt[5]{\frac{8}{9}} : \sqrt[5]{\frac{27}{128}}$
g) $\sqrt[3]{4 \cdot \sqrt[4]{8}}$ h) $\sqrt[5]{9 \cdot \sqrt[3]{3 \cdot \sqrt[2]{27}}}$ i) $\sqrt[6]{6 \cdot \sqrt[4]{6 \cdot \sqrt[3]{6}}}$

7. Vereinfache den Term und stelle ihn in Potenz- und Wurzelschreibweise dar.

a) $\sqrt[4]{a^2 \cdot \sqrt[3]{a^2 \cdot \sqrt{a}}}$ b) $\sqrt[3]{\frac{a^2}{b}} \cdot \sqrt[2]{\frac{a^3}{b}} \cdot \sqrt[4]{\frac{b^3}{a^5}}$
c) $\sqrt[5]{c^4 \cdot \sqrt[3]{c^2}} \cdot \sqrt{c \cdot \sqrt[4]{c^3}} : \sqrt[24]{c^{41}}$ d) $\sqrt[4]{a \cdot \sqrt[6]{ab^2}} \cdot \sqrt[3]{8 \sqrt[4]{a^{11}b^{-1}}} \cdot \sqrt{b^2 \cdot \sqrt[4]{a^2}}$

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1. a) $\sqrt[4]{144} = \sqrt[4]{2^4 \cdot 3^2} = 2 \cdot 3^{\frac{2}{4}} = 2 \cdot 3^{\frac{1}{2}} = 2 \cdot \sqrt{3}$ b) $\sqrt[6]{625} = \sqrt[6]{5^4} = 5^{\frac{4}{6}} = 5^{\frac{2}{3}} = \sqrt[3]{25}$
 c) $\sqrt[9]{125} = \sqrt[9]{5^3} = 5^{\frac{3}{9}} = 5^{\frac{1}{3}} = \sqrt[3]{5}$ d) $\sqrt[10]{32} = \sqrt[10]{2^5} = 2^{\frac{5}{10}} = 2^{\frac{1}{2}} = \sqrt{2}$

2. a) $8^{\frac{1}{6}} = (2^3)^{\frac{1}{6}} = 2^{\frac{3}{6}} = \sqrt{2}$ b) $25^{\frac{1}{8}} = (5^2)^{\frac{1}{8}} = 5^{\frac{2}{8}} = 5^{\frac{1}{4}} = \sqrt[4]{5}$
 c) $27^{\frac{1}{9}} = (3^3)^{\frac{1}{9}} = 3^{\frac{3}{9}} = \sqrt[3]{3}$ d) $16^{\frac{1}{12}} = (2^4)^{\frac{1}{12}} = 2^{\frac{4}{12}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$
 e) $81^{\frac{1}{10}} = (3^4)^{\frac{1}{10}} = 3^{\frac{4}{10}} = 3^{\frac{2}{5}} = \sqrt[5]{9}$ f) $81^{\frac{1}{20}} = (3^4)^{\frac{1}{20}} = 3^{\frac{4}{20}} = 3^{\frac{1}{5}} = \sqrt[5]{3}$
 g) $216^{\frac{1}{6}} = (6^3)^{\frac{1}{6}} = 6^{\frac{3}{6}} = \sqrt{6}$ h) $32^{\frac{1}{15}} = (2^5)^{\frac{1}{15}} = 2^{\frac{5}{15}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$

3. a) $\sqrt[4]{512} = \sqrt[4]{2^9} = 4 \cdot \sqrt[4]{2}$ b) $81^{\frac{1}{3}} = (3^4)^{\frac{1}{3}} = 3^{\frac{4}{3}} = 3^{1+\frac{1}{3}} = 3 \cdot \sqrt[3]{3}$
 c) $\sqrt[3]{250} = \sqrt[3]{2 \cdot 5^3} = 5 \cdot \sqrt[3]{2}$ d) $144^{\frac{3}{5}} = (2^4 \cdot 3^2)^{\frac{3}{5}} = 2^{\frac{12}{5}} \cdot 3^{\frac{6}{5}} = 2^2 \cdot 2^{\frac{2}{5}} \cdot 3^1 \cdot 3^{\frac{1}{5}} = 12 \cdot \sqrt[5]{12}$
 e) $\sqrt[3]{49^2} = \sqrt[3]{7^4} = 7 \cdot \sqrt[3]{7}$ f) $24^{\frac{2}{3}} = (2^3 \cdot 3)^{\frac{2}{3}} = 2^{\frac{2 \cdot 3}{3}} \cdot 3^{\frac{2}{3}} = 4 \cdot \sqrt[3]{9}$
 g) $\sqrt[5]{72^2} = \sqrt[5]{2^6 \cdot 3^4} = 2 \cdot \sqrt[5]{2 \cdot 3^4} = 2 \cdot \sqrt[5]{162}$ h) $8^{\frac{3}{4}} = (2^3)^{\frac{3}{4}} = 2^{\frac{9}{4}} = 2^{2+\frac{1}{4}} = 4 \cdot \sqrt[4]{2}$

4. a) $\sqrt[4]{\frac{1}{8}} = \sqrt[4]{\frac{1 \cdot 2}{2^3 \cdot 2}} = \frac{\sqrt[4]{2}}{2}$ b) $6,75^{\frac{1}{3}} = \sqrt[3]{\frac{27}{4}} = \sqrt[3]{\frac{3^3 \cdot 2}{2^2 \cdot 2}} = \frac{3 \cdot \sqrt[3]{2}}{2}$
 c) $\frac{2}{\sqrt[3]{25}} = \frac{2 \cdot \sqrt[3]{5}}{\sqrt[3]{5^2 \cdot 3 \cdot 5}} = \frac{2 \cdot \sqrt[3]{5}}{5}$ d) $0,25^{\frac{3}{5}} = \sqrt[5]{\left(\frac{1}{4}\right)^3} = \sqrt[5]{\left(\frac{1}{2^2}\right)^3} = \sqrt[5]{\frac{1 \cdot 2^4}{2^6 \cdot 2^4}} = \frac{\sqrt[5]{16}}{4}$
 e) $\frac{2}{\sqrt[4]{9}} = \frac{2 \cdot \sqrt[4]{3^2}}{\sqrt[4]{3^2 \cdot 4 \cdot 3^2}} = \frac{2 \cdot \sqrt[4]{3^2}}{3}$ f) $49^{-\frac{1}{3}} = \frac{1}{7^{\frac{2}{3}}} = \frac{1 \cdot 7^{\frac{1}{3}}}{7^{\frac{2}{3}} \cdot 7^{\frac{1}{3}}} = \frac{7^{\frac{1}{3}}}{7^1} = \frac{\sqrt[3]{7}}{7}$
 g) $\frac{6}{\sqrt[6]{27}} = \frac{6}{3^{\frac{3}{6}}} = \frac{6 \cdot 3^{\frac{1}{2}}}{3^{\frac{3}{2}} \cdot 3^{\frac{1}{2}}} = 2 \cdot \sqrt{3}$ h) $8^{-\frac{3}{4}} = \frac{1}{(2^3)^{\frac{3}{4}}} = \frac{1}{2^{\frac{9}{4}}} = \frac{1 \cdot 2^{\frac{3}{4}}}{2^{\frac{9}{4}} \cdot 2^{\frac{3}{4}}} = \frac{2^{\frac{3}{4}}}{2^3} = \frac{\sqrt[4]{8}}{8}$

5. a) $2 \cdot x^3 = 32 \Leftrightarrow x^3 = 16 \Leftrightarrow x = \sqrt[3]{2^4} = 2 \cdot \sqrt[3]{2}$
 b) $0,2 \cdot x^3 = -32 \Leftrightarrow x^3 = -160 \Leftrightarrow x = -\sqrt[3]{160} \Leftrightarrow x = -2 \cdot \sqrt[3]{20}$
 c) $2 - 3x^4 = 5 \Leftrightarrow 3x^4 = -3 \Leftrightarrow x^4 = -1$ keine Lösung!
 d) $2 \cdot x^3 - 4 = 5 \Leftrightarrow 2 \cdot x^3 = 9 \Leftrightarrow x^3 = \frac{9}{2} \Leftrightarrow x = \sqrt[3]{\frac{9 \cdot 4}{2 \cdot 4}} = \frac{\sqrt[3]{36}}{2}$
 e) $0,25 \cdot x^6 + 4 = 4^{1,5} \Leftrightarrow x^6 + 16 = 4 \cdot 4^{1,5} \Leftrightarrow x^6 = 4 \cdot 8 - 16 \Leftrightarrow x^6 = 16 \Leftrightarrow$
 $x_{1/2} = \pm (2^4)^{\frac{1}{6}} \Leftrightarrow x_{1/2} = \pm 2^{\frac{2}{3}} \Leftrightarrow x_{1/2} = \pm \frac{\sqrt[3]{2}}{2}$
 f) $0,5 \cdot (x-1)^3 - 4 = 12 \Leftrightarrow (x-1)^3 = 16 \cdot 2 \Leftrightarrow x-1 = \sqrt[3]{32} \Leftrightarrow x = 1 + 2 \cdot \sqrt[3]{4}$
 g) $8 \cdot x^{-3} = 5 \Leftrightarrow \frac{8}{5} = x^3 \Leftrightarrow x = \sqrt[3]{\frac{8}{5}} = \sqrt[3]{\frac{2^3 \cdot 5^2}{5 \cdot 5^2}} = \frac{2 \cdot \sqrt[3]{25}}{5}$



$$h) \quad 2 \cdot x^{-5} - 5 = 3^3 \Leftrightarrow 2 \cdot x^{-5} = 32 \Leftrightarrow \frac{1}{x^5} = 16 \Leftrightarrow x^5 = \frac{1}{2^4} \Leftrightarrow x = \sqrt[5]{\frac{1 \cdot 2}{2^4 \cdot 2}} = \frac{\sqrt[5]{2}}{2}$$

$$i) \quad 0,5 \cdot x^{-5} + 11 = 3 \Leftrightarrow \frac{1}{2 \cdot x^5} = -8 \Leftrightarrow -\frac{1}{2 \cdot 8} = x^5 \Leftrightarrow x^5 = -\frac{2}{2^5} \Leftrightarrow x = -\frac{\sqrt[5]{2}}{2}$$

$$j) \quad 2 \cdot x^{\frac{1}{3}} + 4 = 5 \Leftrightarrow x^{\frac{1}{3}} = \frac{1}{2} \Leftrightarrow x = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$k) \quad 0,5 \cdot x^{\frac{2}{3}} + 3 = 5 \Leftrightarrow x^{\frac{2}{3}} = 2 \cdot 2 \Leftrightarrow x = (4)^{\frac{3}{2}} = 2^3 = 8$$

$$l) \quad (x-2)^{\frac{1}{4}} - 3 = 5 \Leftrightarrow (x-2)^{\frac{1}{4}} = 8 \Leftrightarrow x-2 = 8^4 \Leftrightarrow x = 4098$$



$$6. a) \quad 3^{0,5} \cdot 9^{0,75} = 3^{\frac{1}{2}} \cdot 3^{\frac{3}{2}} = 3^2 = 9$$

$$b) \quad 4^{\frac{1}{5}} \cdot 2^{\frac{1}{10}} : 8^{\frac{1}{4}} = 2^{\frac{2}{5}} \cdot 2^{\frac{1}{10}} : 2^{\frac{3}{4}} = 2^{\frac{2}{5} + \frac{1}{10} - \frac{3}{4}} = 2^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{2}} = \frac{\sqrt[4]{8}}{2}$$

$$c) \quad \sqrt[4]{27} \cdot \sqrt[5]{9} \cdot \sqrt[20]{27} = 3^{\frac{3}{4}} \cdot 3^{\frac{2}{5}} \cdot 3^{\frac{3}{20}} = 3^{\frac{3}{4} + \frac{2}{5} + \frac{3}{20}} = 3^{\frac{13}{10}} = 3 \cdot 3^{\frac{3}{10}} = 3 \cdot \sqrt[10]{27}$$

$$d) \quad \sqrt[3]{4} : \sqrt[3]{36} = 2^{\frac{2}{3}} : 6^{\frac{2}{3}} = 2^{\frac{2}{3}} : 6^{\frac{2}{3}} = \frac{2^{\frac{2}{3}} \cdot 6^{\frac{1}{3}}}{6^{\frac{2}{3}} \cdot 6^{\frac{1}{3}}} = \frac{4^{\frac{1}{3}} \cdot 6^{\frac{1}{3}}}{6} = \frac{(2 \cdot 2 \cdot 2 \cdot 3)^{\frac{1}{3}}}{6} = \frac{2 \cdot 3^{\frac{1}{3}}}{6} = \frac{1 \cdot 3^{\frac{1}{3}}}{3} = \frac{\sqrt[3]{3}}{3}$$

$$e) \quad \sqrt[3]{\frac{4}{5}} : \sqrt[6]{25} = \frac{2^{\frac{2}{3}}}{5^{\frac{1}{3}}} : (5^2)^{\frac{1}{6}} = \frac{2^{\frac{2}{3}}}{5^{\frac{1}{3}} \cdot 5^{\frac{2}{6}}} = \frac{2^{\frac{2}{3}}}{5^{\frac{2}{3}}} = \frac{2^{\frac{2}{3}} \cdot 5^{\frac{1}{3}}}{5^{\frac{2}{3}} \cdot 5^{\frac{1}{3}}} = \frac{4^{\frac{1}{3}} \cdot 5^{\frac{1}{3}}}{5} = \frac{20^{\frac{1}{3}}}{5} = \frac{\sqrt[3]{20}}{5}$$

$$f) \quad \sqrt[5]{\frac{8}{9}} : \sqrt[5]{\frac{27}{128}} = \frac{2^{\frac{3}{5}}}{3^{\frac{2}{5}}} : \frac{3^{\frac{3}{5}}}{2^{\frac{7}{5}}} = \frac{2^{\frac{3}{5} + \frac{7}{5}}}{3^{\frac{2}{5} + \frac{3}{5}}} = \frac{2^2}{3^1} = \frac{4}{3}$$

$$g) \quad \sqrt[3]{4 \cdot \sqrt[4]{8}} = \left(2^2 \cdot 2^{\frac{3}{4}}\right)^{\frac{1}{3}} = \left(2^{\frac{11}{4}}\right)^{\frac{1}{3}} = 2^{\frac{11}{12}} = \sqrt[12]{2^{11}} = \sqrt[12]{2048}$$

$$h) \quad \sqrt[5]{9 \cdot \sqrt[3]{3 \cdot \sqrt[2]{27}}} = \left(3^2 \cdot (3 \cdot \sqrt[3]{3})^{\frac{1}{3}}\right)^{\frac{1}{5}} = \left(3^2 \cdot (3^{\frac{5}{3}})^{\frac{1}{3}}\right)^{\frac{1}{5}} = \left(3^{2 + \frac{5}{6}}\right)^{\frac{1}{5}} = 3^{\frac{17}{6} \cdot \frac{1}{5}} = 3^{\frac{17}{30}} = \sqrt[30]{3^{17}}$$

$$i) \quad \sqrt[6]{6 \cdot \sqrt[4]{6 \cdot \sqrt[3]{6}}} = \left(6 \cdot (6 \cdot 6^{\frac{1}{3}})^{\frac{1}{4}}\right)^{\frac{1}{6}} = \left(6 \cdot (6^{\frac{4}{3}})^{\frac{1}{4}}\right)^{\frac{1}{6}} = \left(6^{1 + \frac{1}{3}}\right)^{\frac{1}{6}} = 6^{\frac{4}{3 \cdot 6}} = 6^{\frac{2}{9}} = \sqrt[9]{36}$$

$$7. a) \quad \sqrt[4]{a^2 \cdot \sqrt[3]{a^2 \cdot \sqrt{a}}} = \left(a^2 \cdot (a^{2 + \frac{1}{2}})^{\frac{1}{3}}\right)^{\frac{1}{4}} = \left(a^2 \cdot a^{\frac{5}{6}}\right)^{\frac{1}{4}} = \left(a^{\frac{17}{6}}\right)^{\frac{1}{4}} = a^{\frac{17}{24}} = \sqrt[24]{a^{17}}$$

$$b) \quad \sqrt[3]{\frac{a^2}{b} \cdot \sqrt[2]{\frac{a^3}{b}}} \cdot \sqrt[4]{\frac{b^3}{a^5}} = \left(a^2 \cdot b^{-1} \cdot a^{\frac{3}{2}} \cdot b^{-\frac{1}{2}} \cdot b^{\frac{3}{4}} \cdot a^{-\frac{5}{4}}\right)^{\frac{1}{3}} = \left(a^{\frac{9}{4}} \cdot b^{-\frac{3}{4}}\right)^{\frac{1}{3}} = a^{\frac{3}{4}} \cdot b^{-\frac{1}{4}} = \frac{\sqrt[4]{a^3}}{\sqrt[4]{b}} = \frac{\sqrt[4]{a^3 b^3}}{b}$$

$$c) \quad \sqrt[5]{c^4 \cdot \sqrt[3]{c^2}} \cdot \sqrt{c \cdot \sqrt[4]{c^3}} : \sqrt[24]{c^{41}} = \left(c^4 \cdot c^{\frac{2}{3}}\right)^{\frac{1}{5}} \cdot \left(c \cdot c^{\frac{3}{4}}\right)^{\frac{1}{2}} : c^{\frac{41}{24}} = c^{\frac{14}{3 \cdot 5}} \cdot c^{\frac{7}{4 \cdot 2}} \cdot c^{-\frac{41}{24}} = c^{\frac{1}{10}} = \sqrt[10]{c}$$

$$d) \quad \sqrt[4]{a \cdot \sqrt[6]{ab^2}} \cdot \sqrt[3]{8 \sqrt[8]{a^{11}b^{-1}}} \cdot \sqrt{b^2 \cdot \sqrt[4]{a^2}} = \left(a \cdot a^{\frac{1}{6}} \cdot b^{\frac{2}{6}}\right)^{\frac{1}{4}} \cdot \left((a^{11}b^{-1})^{\frac{1}{8}}\right)^{\frac{1}{3}} \cdot \left(b^2 \cdot a^{\frac{2}{4}}\right)^{\frac{1}{2}} = ab \cdot \sqrt[4]{a^3} \cdot \sqrt[24]{b}$$