

Mathematik * Jahrgangsstufe 9 * Binomische Formeln und Wurzeln



Die drei **binomischen Formeln** lauten:

$$(a + b)^2 = a^2 + 2ab + b^2 \quad ; \quad (a - b)^2 = a^2 - 2ab + b^2 \quad ; \quad (a + b) \cdot (a - b) = a^2 - b^2$$

1. Ergänze so, dass man den Term als Produkt schreiben kann.

a) $\frac{1}{4}x^2 + \square + 4 = (\quad)^2$ b) $9a^2 - 24ab + \square = (\quad)^2$
c) $2,25x^2 + \square + \frac{1}{4} = (\quad)^2$ d) $1,96a^2 - 14ab + \square = (\quad)^2$
e) $2x^2 + \square + 4 = (\quad)^2$ f) $3a^2 - 2\sqrt{6}ab + \square = (\quad)^2$

2. Schreibe – falls möglich – den Term ohne Wurzel.

a) $\sqrt{4x^2 + 2xy + 0,25y^2}$ b) $\sqrt{36x^2 + 4y^2 - 24xy}$
c) $\sqrt{\frac{1}{4}x^2 + y^2 - xy}$ d) $\sqrt{4 - 6a + 2,25a^2}$
e) $\sqrt{9x^2 + 18xy + 4y^2}$ f) $\sqrt{9x^2 - 9x + 2,25}$



3. Vereinfache.

a) $\frac{\sqrt{2}}{1 - \sqrt{2}}$ b) $\frac{\sqrt{2}}{\sqrt{3}} + \frac{1}{\sqrt{6}}$ c) $\frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}}$
d) $\frac{\sqrt{2x^2 - 8}}{\sqrt{x + 2}}$ e) $\frac{\sqrt{4xy}}{\sqrt{x} - \sqrt{y}}$ f) $\frac{\sqrt{8x}}{\sqrt{2x} - 2}$

4. Vereinfache so weit wie möglich.

a) $\sqrt{3x^2 - 12xy + 12y^2}$ b) $\sqrt{0,5x^2 + 4,5y^2 - 3xy}$
c) $\sqrt{\frac{x^2 - 4x + 4}{4x - 8}}$ d) $\frac{0,25a^2 + ab + b^2}{4a + 8b}$
e) $\frac{1}{\sqrt{2}} - \frac{2}{\sqrt{5}}$ f) $\frac{3 + \sqrt{3}x + 0,25x^2}{x + 2\sqrt{3}}$



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Lösungen

1. a) $\frac{1}{4}x^2 + \boxed{2x} + 4 = \left(\frac{1}{2}x + 2\right)^2$ b) $9a^2 - 24ab + \boxed{16b^2} = (3a - 4b)^2$

c) $2,25x^2 + \boxed{1,5x} + \frac{1}{4} = \left(1,5x + \frac{1}{2}\right)^2$ d) $1,96a^2 - 14ab + \boxed{25b^2} = (1,4a - 5b)^2$

e) $2x^2 + \boxed{4\sqrt{2}x} + 4 = (\sqrt{2}x + 2)^2$ f) $3a^2 - 2\sqrt{6}ab + \boxed{2b^2} = (\sqrt{3}a - \sqrt{2}b)^2$

2. a) $\sqrt{4x^2 + 2xy + 0,25y^2} = \sqrt{(2x + 0,5y)^2} = |2x + 0,5y|$

b) $\sqrt{36x^2 + 4y^2 - 24xy} = \sqrt{(6x - 2y)^2} = |6x - 2y|$

c) $\sqrt{\frac{1}{4}x^2 + y^2 - xy} = \sqrt{\left(\frac{1}{2}x - y\right)^2} = \left|\frac{1}{2}x - y\right|$

d) $\sqrt{4 - 6a + 2,25a^2} = \sqrt{(1,5a - 2)^2} = |1,5a - 2|$

e) $\sqrt{9x^2 + 18xy + 4y^2}$ lässt sich nicht ohne Wurzel schreiben!

f) $\sqrt{9x^2 - 9x + 2,25} = \sqrt{(3x - 1,5)^2} = |3x - 1,5|$



2. a) $\frac{\sqrt{2}}{1 - \sqrt{2}} = \frac{\sqrt{2} \cdot (1 + \sqrt{2})}{(1 - \sqrt{2}) \cdot (1 + \sqrt{2})} = \frac{\sqrt{2} + 2}{1 - 2} = -2 - \sqrt{2}$

b) $\frac{\sqrt{2}}{\sqrt{3}} + \frac{1}{\sqrt{6}} = \frac{\sqrt{2} \cdot \sqrt{6} + \sqrt{3}}{\sqrt{3} \cdot \sqrt{6}} = \frac{2 \cdot \sqrt{3} + \sqrt{3}}{3 \cdot \sqrt{2}} = \frac{3 \cdot \sqrt{3}}{3 \cdot \sqrt{2}} = \frac{\sqrt{3} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{6}}{2}$

c) $\frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}} = \frac{\sqrt{6} \cdot (\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2}) \cdot (\sqrt{3} - \sqrt{2})} = \frac{3\sqrt{2} - 2\sqrt{3}}{3 - 2} = 3\sqrt{2} - 2\sqrt{3}$

d) $\frac{\sqrt{2x^2 - 8}}{\sqrt{x + 2}} = \frac{\sqrt{2(x^2 - 4) \cdot (x + 2)}}{\sqrt{x + 2} \cdot \sqrt{x + 2}} = \frac{\sqrt{2(x - 2) \cdot (x + 2) \cdot (x + 2)}}{|x + 2|} = \sqrt{2(x - 2)}$

e) $\frac{\sqrt{4xy}}{\sqrt{x} - \sqrt{y}} = \frac{\sqrt{4xy} \cdot (\sqrt{x} + \sqrt{y})}{(\sqrt{x} - \sqrt{y}) \cdot (\sqrt{x} + \sqrt{y})} = \frac{2x\sqrt{y} + 2y\sqrt{x}}{x - y}$

f) $\frac{\sqrt{8x}}{\sqrt{2x} - 2} = \frac{2\sqrt{2x} \cdot (\sqrt{2x} + 2)}{(\sqrt{2x} - 2) \cdot (\sqrt{2x} + 2)} = \frac{4x + 4\sqrt{2x}}{2x - 4} = \frac{2x + 2\sqrt{2x}}{x - 2}$

$$3. \text{ a) } \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2} \cdot (1+\sqrt{2})}{(1-\sqrt{2}) \cdot (1+\sqrt{2})} = \frac{\sqrt{2}+2}{1-2} = -2-\sqrt{2}$$

$$\text{ b) } \frac{\sqrt{2}}{\sqrt{3}} + \frac{1}{\sqrt{6}} = \frac{\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} + \frac{1 \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = \frac{\sqrt{6}}{3} + \frac{\sqrt{6}}{6} = \frac{2 \cdot \sqrt{6} + \sqrt{6}}{6} = \frac{3\sqrt{6}}{6} = 0,5 \cdot \sqrt{6}$$

$$\text{ c) } \frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}} = \frac{\sqrt{6} \cdot (\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2}) \cdot (\sqrt{3} - \sqrt{2})} = \frac{\sqrt{18} - \sqrt{12}}{3-2} = 3\sqrt{2} - 2\sqrt{3}$$

$$\text{ d) } \frac{\sqrt{2x^2 - 8}}{\sqrt{x+2}} = \frac{\sqrt{2(x^2 - 2^2)}}{\sqrt{x+2}} = \frac{\sqrt{2(x-2) \cdot (x+2)}}{\sqrt{x+2}} = \frac{\sqrt{2(x-2)} \cdot \sqrt{x+2}}{\sqrt{x+2}} = \sqrt{2(x-2)}$$

$$\text{ e) } \frac{\sqrt{4xy}}{\sqrt{x-y}} = \frac{\sqrt{4xy} \cdot (\sqrt{x} + \sqrt{y})}{(\sqrt{x} - \sqrt{y}) \cdot (\sqrt{x} + \sqrt{y})} = \frac{\sqrt{4x^2y} + \sqrt{4xy^2}}{x-y} = \frac{2x \cdot \sqrt{y} + 2y \cdot \sqrt{x}}{x-y}$$

$$\text{ f) } \frac{\sqrt{8x}}{\sqrt{2x-2}} = \frac{\sqrt{8x} \cdot (\sqrt{2x+2})}{(\sqrt{2x-2}) \cdot (\sqrt{2x+2})} = \frac{\sqrt{16x^2} + \sqrt{16x}}{2x-4} = \frac{4x + 4\sqrt{x}}{2 \cdot (x-2)} = \frac{2x + 2\sqrt{x}}{x-2}$$

$$4. \text{ a) } \sqrt{3x^2 - 12xy + 12y^2} = \sqrt{3 \cdot (x^2 - 4xy + 4y^2)} = \sqrt{3 \cdot (x-2y)^2} = |x-2y| \cdot \sqrt{3}$$

$$\text{ b) } \sqrt{0,5x^2 + 4,5y^2 - 3xy} = \sqrt{0,5 \cdot (x^2 - 6xy + 9y^2)} = \sqrt{\frac{2 \cdot (x-3y)^2}{4}} = \frac{|x-3y| \cdot \sqrt{2}}{2}$$

$$\text{ c) } \sqrt{\frac{x^2 - 4x + 4}{4x - 8}} = \sqrt{\frac{(x-2)^2}{4(x-2)}} = \sqrt{\frac{(x-2)}{4}} = \frac{\sqrt{x-2}}{2}$$

$$\text{ d) } \frac{0,25a^2 + ab + b^2}{4a + 8b} = \frac{0,25 \cdot (a^2 + 4ab + 4b^2)}{4 \cdot (a+2b)} = \frac{0,25 \cdot (a+2b)^2}{4 \cdot (a+2b)} = \frac{a+2b}{16}$$

$$\text{ e) } \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{5}} = \frac{\sqrt{2}}{2} - \frac{2 \cdot \sqrt{5}}{5} = \frac{5\sqrt{2} - 4\sqrt{5}}{10} = 0,5\sqrt{2} - 0,4\sqrt{5}$$

$$\text{ f) } \frac{3 + \sqrt{3}x + 0,25x^2}{x + 2\sqrt{3}} = \frac{(\sqrt{3} + 0,5x)^2}{2 \cdot (0,5x + \sqrt{3})} = \frac{1}{2} \cdot (\sqrt{3} + 0,5x)$$

